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Attitude Maneuver of Dual Tethered Satellite Platforms Through Tether Offset Change

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A new concept is presented that involves the use of dual satellite platforms connected through a tether. The feasibility of suitably varying tether offsets for achieving desired maneuver of both the platforms is explored. The Lagrangian formulation approach is utilized to develop the governing system of nonlinear ordinary differential equations for the constrained system. A simple open-loop strategy is developed for the tether offset variations, that ensures judiciously controlled changes in the orientation of satellite platforms. The numerical simulation of the nonlinear governing equations of motion for these tether offset variations establishes the feasibility of achieving desired attitude maneuvers. The nearly passive nature of the proposed orientation control strategy makes it particularly attractive for future space missions.

Nomenclature

a_{xi}, a_{yi}, a_{zi}	=	x_i, y_i, z_i coordinates of the offset points of tether in the S–$x_i y_i z_i$ frame in satellite platform i, m
$\hat{a}_{ki}$	=	a_{ki}/L_{ref}, where $k = x, y, z$
$(a_{ki})_f$	=	tether offsets for final orientation angles of satellite platform i, where $k = x, y, z$, m
$(a_{ki})_0$	=	tether offsets for initial orientation of satellite platform i, where $k = x, y, z$, m
$(a_{ki})_\theta$	=	tether offsets on platform i at θ, where $k = x, y, z$, m
$(\hat{a}_{ki})_f$	=	$(a_{ki})_f/L_{\mathrm{ref}}$, where $k = x, y, z$
$(\hat{a}_{ki})_0$	=	$(a_{ki})_0/L_{\mathrm{ref}}$, where $k = x, y, z$
$(\hat{a}_{ki})_\theta$	=	$(a_{ki})_\theta/L_{\mathrm{ref}}$, where $k = x, y, z$
C	=	$EA/(m_2 L_{\mathrm{ref}} \Omega^2)$
EA	=	tether modulus of rigidity, N
f	=	platform mass ratio, m_1/m_2
g	=	platform moment of inertia ratio, I_{x1}/I_{x2}
I_{ki}	=	principal centroidal moments of inertia about k_i axis for satellite platform i, where $k = x, y, z$ and $i = 1, 2$, kg · m^2
K_{i1}	=	mass distribution parameter for satellite platform i $(I_{xi} - I_{yi})/I_{zi}$
K_{i2}	=	mass distribution parameter for satellite platform i $(I_{xi} - I_{zi})/I_{yi}$
K_{i+2}	=	$1 - K_{i1}(K_{i2} - 1)/(K_{i1}K_{i2} - 1)$
K_{i+4}	=	$(K_{i2} - 1)/(K_{i1}K_{i2} - 1)$
K_1	=	K_{11} or K_{21}, when $K_{11} = K_{21}$
K_2	=	K_{12} or K_{22}, when $K_{12} = K_{22}$
L	=	distance between two platform mass centers, m
$\boldsymbol{L}$	=	position vector of center of mass of satellite platform 2 with respect to center of mass of satellite platform 1
L_e	=	L at system equilibrium, m
L_{ref}	=	reference length $(I_{x1}/m_2)^{1/2}$, m
L_t	=	stretched tether length, m
L_{t0}	=	unstreched tether length, m
L_0	=	L when tether strain is zero, m
$(L_{t0})_{\mathrm{cr}}$	=	critical unstreched tether length, m
l	=	L/L_{ref}
l_t	=	L_t/L_{ref}
l_{t0}	=	L_{t0}/L_{ref}
l_0	=	L_0/L_{ref}
l_0'	=	l' at $\theta = 0$
$(l_0)_{\mathrm{cr}}$	=	$(L_{t0})_{\mathrm{cr}}/L_{\mathrm{ref}}$
m_i	=	mass of satellite platform i, kg
R	=	orbital radius, m
r_{bi}	=	radial tether offset $S_i E_i$ at satellite platform i from its mass center S_i, Fig. 2, m
$\hat{r}_{bi}$	=	r_{bi}/L_{ref}
$\hat{r}_{b1}$	=	$-\hat{r}_{b2}$ when $\lvert\hat{r}_{b1}\rvert = \lvert\hat{r}_{b2}\rvert$
S–$x_r y_r z_r$	=	coordinate axes for relative motion of dual satellite platform
S–$x_0 y_0 z_0$	=	coordinate axes in the local vertical frame
S_i–$x_i y_i z_i$	=	body coordinate for satellite platform i
T	=	kinetic energy of the system
$U(\varepsilon_t)$	=	1 for $\varepsilon_t \geq 0$ and 0 for $\varepsilon_t < 0$
V	=	potential energy of the system
$\alpha_i, \phi_i, \gamma_i$	=	pitch, roll and yaw angles, respectively, for satellite platform i, deg
$\alpha_{ie}, \phi_{ie}, \gamma_{ie}$	=	equilibrium pitch, roll and yaw angles, respectively, for satellite platform i, deg
$\alpha_{i0}, \phi_{i0}, \gamma_{i0}$	=	α_i, ϕ_i, and γ_i at $\theta = 0$, deg
$\alpha_{i0}', \phi_{i0}', \gamma_{i0}'$	=	α_i', ϕ_i', and γ_i' at $\theta = 0$
β, η	=	relative in-plane and out-of-plane swing angles of $\boldsymbol{L}$, respectively, deg
β_e, η_e	=	β and η at system equilibrium, deg
β_0, η_0	=	β and η at $\theta = 0$, deg
β_0', η_0'	=	β' and η' at $\theta = 0$
ε_t	=	tether strain
θ	=	in-plane angle measured relative to a specified reference line, deg
λ	=	Lagrange multiplier
μ	=	Earth's gravitational constant, m^3/s^2
τ	=	number of orbits desired for completing the required tether offset changes
φ	=	tether length constraint function
Ω	=	angular velocity $(\mu/R^3)^{1/2}$, rad/s

Subscripts

i	=	satellite platform i, 1, 2
0	=	$\theta = 0$

Superscripts

$'$, $''$	=	d( )/ dθ and d^2( )/ dθ^2, respectively

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Introduction

THE advent of tethered satellite systems (TSS)[1] marks the beginning of a new era in space research. Several interesting space applications of tethers[2] have been proposed and analyzed. The earlier methods suggested for ensuring satellite librational stability through a single tether–mass attachment were based on feedback control of the control moment by regulating tether lengths or offsets alone or in combination with established active control devices.[3–7] Recently, Kumar,[8] Kumar and Kumar,[9–12] and Kumar[13] established the feasibility of using TSS to achieve nearly passive satellite pointing stability, as well as attitude maneuver. The attitude maneuver of a platform[9,13] was accomplished through change of tether lengths carrying a subsatellite in a pendulum fashion. Modi et al.[14] have studied tether offset variation approach for the orientation control of the main satellite. However, this approach involves a long tether length as well because it can only accomplish the orientation control of the main satellite. To circumvent these limitations, we present a new concept involving the use of dual satellite platforms connected through a single tether. The feasibility of suitably varying tether offsets for achieving desired attitude maneuver of both the platforms is explored. With a short tether needed for the proposed system, the associated problems of tether deployment as well as chances of tether being cut by debris attack may be considerably reduced.

The Lagrangian formulation procedure is utilized to obtain the governing ordinary differential equations of motion for the proposed constrained system moving in a circular orbit. It is assumed that the tether dynamics do not effect the orbital dynamics for the relatively short tether length considered here. A simple open-loop control law for varying the tether offsets is developed to achieve desired orientations of the platforms. Finally, for a detailed assessment of the proposed attitude maneuver strategy, the set of exact governing equations of motion is numerically integrated.

Problem Formulation

The investigation is initiated by formulating the equations of motion of the proposed system. The system model comprises a tether connecting platform 1 at a point below its mass center with zero yaw-plane offset to platform 2 at a point above its mass center with zero yaw-plane offset (Fig. 1).

The tether is assumed to be made of a light but rigid material such as Kevlar® and, hence, is taken to have negligible mass. The tether's transverse and torsional vibrations are ignored. The coordinate frame $x_0 y_0 z_0$ passing through the system center of mass S represents the orbital reference frame. The x_0 axis is taken normal to the orbital plane, y_0 axis points along the local vertical, and z_0 axis completes the right-hand triad. The three reference frames are used to specify the motion of the TSS relative to this local orbital frame. The orientation of the satellite platform i, $i = 1, 2$, is specified by a set of three successive rotations: α_i (pitch) about the x_i axis, γ_i about the new yaw axis, and finally ϕ_i about the resulting roll axis. The corresponding principal body-fixed coordinate axes for platforms 1 and 2 are S_1–$x_1 y_1 z_1$ and S_2–$x_2 y_2 z_2$, respectively. Similarly, for the variable length vector $\boldsymbol{L}$ joining the two platform mass centers S_1 and S_2, angle β denotes rotation about the axis normal to the orbital plane and is referred to as in-plane swing angle, whereas, angle η represents its out-of-plane swing angle. The resulting coordinate frame associated with this vector is S–$x_r y_r z_r$.

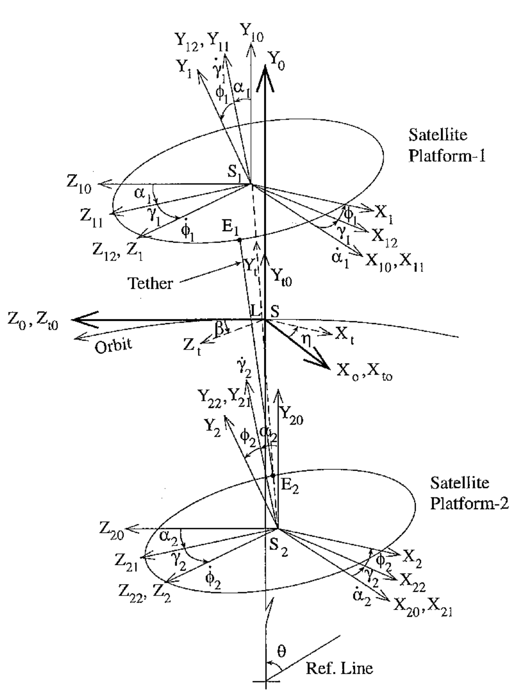

Fig. 1 Geometry of a dual tethered satellite platform undergoing three-dimensional librations.

For convenience of system representation and response simulation, the governing relations are expressed in dimensionless form. The system under consideration has 10 generalized coordinates: six coordinates for platform rotations (pitch α_i, roll ϕ_i, and yaw γ_i, $i = 1, 2$), three coordinates for vector $\boldsymbol{L}$ (length L, β, and η), and tether strain (ε_t).

The preceding generalized variables are not independent and are related through dimensionless tether length constraint as follows:

$$\varphi = l_t - \Big\{\hat{a}_{x1}^2 + \hat{a}_{y1}^2 + \hat{a}_{z1}^2 + (l\sin\eta + h_2)^2 + (-l\cos\beta\cos\eta + r_2)^2 + (-l\sin\beta\cos\eta + s_2)^2 - 2[h_1(l\sin\eta + h_2) + r_1(-l\cos\beta\cos\eta + r_2) + s_1(-l\sin\beta\cos\eta + s_2)]\Big\}^{\frac{1}{2}} = 0 \tag{1}$$

where

$$l_t = l_{t0}(1 + \varepsilon_t)$$

$$h_i = \hat{a}_{zi}\sin\gamma_i + \hat{a}_{xi}\cos\gamma_i\cos\phi_i - \hat{a}_{yi}\sin\phi_i\cos\gamma_i, \qquad i = 1, 2$$

$$r_i = -\hat{a}_{zi}\sin\alpha_i\cos\gamma_i + \hat{a}_{xi}(\cos\alpha_i\sin\phi_i + \sin\alpha_i\sin\gamma_i\cos\phi_i) + \hat{a}_{yi}(\cos\alpha_i\cos\phi_i - \sin\alpha_i\sin\gamma_i\sin\phi_i), \qquad i = 1, 2$$

$$s_i = \hat{a}_{zi}\cos\alpha_i\cos\gamma_i + \hat{a}_{xi}(\sin\alpha_i\sin\phi_i - \cos\alpha_i\sin\gamma_i\cos\phi_i) + \hat{a}_{yi}(\sin\alpha_i\cos\phi_i + \cos\alpha_i\sin\gamma_i\sin\phi_i), \qquad i = 1, 2$$

To apply the Lagrangian approach for the formulation of the equations of motion, the expressions for the system kinetic energy T, as well as the potential energy V are first obtained:

$$T = \frac{1}{2}(m_1 + m_2)\dot{\theta}^2 R^2 + \frac{1}{2}\left(\frac{m_2}{1 + 1/f}\right)\{\dot{L}^2 + [(\dot{\theta} + \dot{\beta})^2\cos^2\eta + \dot{\eta}^2]L^2\} + \frac{1}{2}\sum_{i=1}^{2}\Big\{I_{xi}[(\dot{\theta} + \dot{\alpha}_i)\cos\phi_i\cos\gamma_i + \dot{\gamma}_i\sin\phi_i]^2 + I_{yi}[-(\dot{\theta} + \dot{\alpha}_i)\sin\phi_i\cos\gamma_i + \dot{\gamma}_i\cos\phi_i]^2 + I_{zi}[-(\dot{\theta} + \dot{\alpha}_i)\sin\gamma_i + \dot{\phi}_i]^2\Big\}$$

$$V = \sum_{i=1}^{2}\Bigg\{-\frac{\mu}{R}m_i + \frac{1}{4}\frac{\mu}{R^3}\Big[(I_{xi} + I_{yi} + I_{zi}) - 3\Big\{(I_{yi} + I_{zi} - I_{xi})(\cos\alpha_i\sin\phi_i + \sin\alpha_i\cos\phi_i\sin\gamma_i)^2 + (I_{xi} + I_{zi} - I_{yi})(\cos\alpha_i\cos\phi_i - \sin\alpha_i\sin\phi_i\sin\gamma_i)^2 + (I_{xi} + I_{yi} - I_{zi})\sin^2\alpha_i\cos^2\gamma_i\Big\}\Big] + \frac{1}{2}\frac{\mu}{R^3}\left(\frac{m_2}{1 + 1/f}\right) \times (1 - 3\cos^2\beta\cos^2\eta)L^2 + \frac{1}{2}\mathrm{EA}L_{t0}\varepsilon_t^2 U(\varepsilon_t)\Bigg\}$$

The term $U(\varepsilon_t)$ in the potential energy expression is simply a unit function, the use of which precludes any negative strain in the tether.

The Lagrangian equations of motion corresponding to the various generalized coordinates indicated earlier may be obtained using the general relation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = Q + \lambda\frac{\partial \varphi}{\partial q}$$

where q is the generalized coordinate, Q the generalized force corresponding to the generalized coordinate q, and λ the Lagrange multiplier corresponding to the constraint as indicated in Eq. (1).

Substituting the generalized coordinates in the preceding equations and carrying out the algebraic manipulation and nondimensionalization, we get the following governing nonlinear, coupled ordinary differential equations of motion in the dimensionless form.

Pitch (α_i), $i = 1, 2$:

$$\begin{aligned}
&\left\{\left[1 + (K_{i+2} - 1)\sin^2\phi_i\right]\cos^2\gamma_i + K_{i+4}\sin^2\gamma_i\right\}\alpha_i'' + K_{i+4}\phi_i'' \\
&\quad\times \sin\gamma_i - \left(\frac{1}{2}\right)(K_{i+2} - 1)\gamma_i''\sin 2\phi_i\cos\gamma_i + (1 + \alpha_i') \\
&\quad\times \left\{(K_{i+2} - 1)\phi_i'\sin 2\phi_i\cos^2\gamma_i - \left[1 + (K_{i+2} - 1)\sin^2\phi_i\right.\right. \\
&\quad\left.\left. - K_{i+4}\right]\gamma_i'\sin 2\gamma_i\right\} + \gamma_i'[(1 - K_{i+2})\phi_i'\cos 2\phi_i\cos\gamma_i \\
&\quad - (1 - K_{i+2})\gamma_i'\sin\phi_i\cos\phi_i\sin\gamma_i + K_{i+4}\phi_i'\cos\gamma_i] \\
&\quad - \left(\frac{3}{2}\right)\left[(K_{i+2} + K_{i+4} - 1)(\cos\alpha_i\sin\phi_i + \sin\alpha_i\sin\gamma_i\right. \\
&\quad\times \cos\phi_i)(-\sin\alpha_i\sin\phi_i + \cos\alpha_i\cos\phi_i\sin\gamma_i) + (1 + K_{i+2} \\
&\quad - K_{i+4})\sin\alpha_i\cos\alpha_i\cos^2\gamma_i - (1 - K_{i+2} + K_{i+4}) \\
&\quad\times (\cos\alpha_i\cos\phi_i - \sin\alpha_i\sin\gamma_i\sin\phi_i)(\sin\alpha_i\cos\phi_i \\
&\quad\left. + \cos\alpha_i\sin\gamma_i\sin\phi_i)\right] - p\lambda\left(\frac{\partial\varphi}{\partial\alpha_i}\right) = 0 \qquad (2)
\end{aligned}$$

Yaw (γ_i), $i = 1, 2$:

$$\begin{aligned}
&[(1 - K_{i+2})\sin\phi_i\cos\phi_i\cos\gamma_i]\alpha_i'' + \left[1 + (K_{i+2} - 1)\cos^2\phi_i\right]\gamma_i'' \\
&\quad + \left(\frac{1}{2}\right)(1 + \alpha_i')(1 - K_{i+2})[2\phi_i'\cos 2\phi_i\cos\gamma_i - \gamma_i'\sin 2\phi_i \\
&\quad\times \sin\gamma_i] + (1 - K_{i+2})\gamma_i'\phi_i'\sin 2\phi_i + (1 + \alpha_i')[(1 + \alpha_i')\cos\phi_i \\
&\quad\times \cos\gamma_i + \gamma_i'\sin\phi_i]\cos\phi_i\sin\gamma_i + K_{i+2}[(1 + \alpha_i')\sin\phi_i\cos\gamma_i \\
&\quad - \gamma_i'\cos\phi_i](1 + \alpha_i')\sin\phi_i\sin\gamma_i - K_{i+4}(1 + \alpha_i')[(1 + \alpha_i') \\
&\quad\times \sin\gamma_i + \phi_i']\cos\gamma_i - \left(\frac{3}{2}\right)[-(K_{i+2} + K_{i+4} - 1) \\
&\quad\times (\cos\alpha_i\sin\phi_i + \sin\alpha_i\sin\gamma_i\cos\phi_i)\sin\alpha_i\cos\gamma_i\cos\phi_i \\
&\quad + (1 + K_{i+2} - K_{i+4})\sin^2\alpha_i\sin\gamma_i\cos\gamma_i + (1 - K_{i+2} \\
&\quad + K_{i+4})(\cos\alpha_i\cos\phi_i - \sin\alpha_i\sin\gamma_i\sin\phi_i) \\
&\quad\times \sin\alpha_i\cos\gamma_i\sin\phi_i] - p\lambda\left(\frac{\partial\varphi}{\partial\gamma_i}\right) = 0 \qquad (3)
\end{aligned}$$

Roll (ϕ_i), $i = 1, 2$:

$$\begin{aligned}
&K_{i+4}\alpha_i''\sin\gamma_i + K_{i+4}(1 + \alpha_i')\gamma_i'\cos\gamma_i + K_{i+4}\phi_i'' + [(1 + \alpha_i') \\
&\quad\times \sin\phi_i\cos\gamma_i - \gamma_i'\cos\phi_i][(1 + \alpha_i')\cos\phi_i\cos\gamma_i + \gamma_i'\sin\phi_i] \\
&\quad - K_{i+2}[(1 + \alpha_i')\sin\phi_i\cos\gamma_i - \gamma_i'\cos\phi_i][(1 + \alpha_i')\cos\phi_i\cos\gamma_i \\
&\quad + \gamma_i'\sin\phi_i] - \left(\frac{3}{2}\right)[(K_{i+2} + K_{i+4} - 1)(\cos\alpha_i\sin\phi_i \\
&\quad + \sin\alpha_i\sin\gamma_i\cos\phi_i)(\cos\alpha_i\cos\phi_i - \sin\alpha_i\sin\gamma_i\sin\phi_i) \\
&\quad - (1 - K_{i+2} + K_{i+4})(\cos\alpha_i\cos\phi_i - \sin\alpha_i\sin\gamma_i\sin\phi_i) \\
&\quad\times (\cos\alpha_i\sin\phi_i + \sin\alpha_i\sin\gamma_i\cos\phi_i)] - p\lambda\left(\frac{\partial\varphi}{\partial\phi_i}\right) = 0 \qquad (4)
\end{aligned}$$

L, in-plane swing (β):

$$\begin{aligned}
&\beta'' + 2(1 + \beta')\left[\left(\frac{l'}{l}\right) - \eta'\tan\eta\right] + 3\sin\beta\cos\beta \\
&\quad - \left[\frac{1}{(l\cos\eta)^2}\right]\left(1 + \frac{1}{f}\right)\lambda\left(\frac{\partial\varphi}{\partial\beta}\right) = 0 \qquad (5)
\end{aligned}$$

L, out-of-plane swing (η):

$$\begin{aligned}
&\eta'' + 2\eta'\left(\frac{l'}{l}\right) + (1 + \beta')^2\sin\eta\cos\eta + 3\sin\eta\cos\eta\cos^2\beta \\
&\quad - \left(\frac{1}{l^2}\right)\left(1 + \frac{1}{f}\right)\lambda\left(\frac{\partial\varphi}{\partial\eta}\right) = 0 \qquad (6)
\end{aligned}$$

L, nondimensional length l:

$$\begin{aligned}
&l'' - [\eta'^2 + (1 + \beta')^2\cos^2\eta]l + (1 - 3\cos^2\beta\cos^2\eta)l \\
&\quad - \left(1 + \frac{1}{f}\right)\lambda\left(\frac{\partial\varphi}{\partial l}\right) = 0 \qquad (7)
\end{aligned}$$

Tether strain ε_t:

$$\lambda = C\varepsilon_t U(\varepsilon_t) \qquad (8)$$

where $p = 1$ if $i = 1$, $p = g$ if $i = 2$, and the independent variable is longitude angle θ.

Development of Open-Loop Tether Offset Control Laws

The equations of motion of the proposed system are quite complex, and it does not appear possible to obtain a solution through an analytical approach. Because we desire variable changes in platform orientation, it would be necessary to first examine these governing equations of motion for feasible platform equilibrium configurations and their likely dependence on system parameters. Here, an attempt is made to develop open-loop tether offset control laws to achieve desired orientation of platforms.

For platforms positioned in an arbitrary steady state equilibrium configuration given, for example, by $\alpha_i = \alpha_{ie}$, $\phi_i = \phi_{ie}$, $\gamma_i = \gamma_{ie}$, $\beta = \beta_e$, $\eta = \eta_e$, and $l = l_e$, a substitution of the steady-state conditions

$$\alpha_i = \alpha_{ie}, \qquad \alpha_i' = \alpha_i'' = 0, \qquad \phi_i = \phi_{ie}, \qquad \phi_i' = \phi_i'' = 0$$

$$\gamma_i = \gamma_{ie}, \qquad \gamma_i' = \gamma_i'' = 0, \qquad i = 1, 2$$

$$\beta = \beta_e, \qquad \beta' = \beta'' = 0, \qquad \eta = \eta_e, \qquad \eta' = \eta'' = 0$$

$$l = l_e, \qquad l' = l'' = 0$$

into the Eqs. (2–8) and the consideration that $l_e \gg |\hat{a}_{mi}|$, $m = x, y, z$; $i = 1, 2$, lead to the following relations:

$$\begin{aligned}
\beta_e &= \left(\tfrac{1}{2}\right)\sin^{-1}\left\{(1 + 1/f)\left[1 + (K_3 - 1)\cos^2\phi_{1e} - K_5\right]\right. \\
&\quad\times\left[\sin(2\alpha_{1e})/l_e^2\right] + (1/g)(1 + 1/f) \\
&\quad\left.\times\left[1 + (K_4 - 1)\cos^2\phi_{2e} - K_6\right]\left[\sin(2\alpha_{2e})/l_e^2\right]\right\} \\
\eta_e &= \left(\tfrac{1}{2}\right)\sin^{-1}\left\{\left[(K_3 - 1)\left(1 + 3\cos^2\alpha_{1e}\right)\sin(2\phi_{1e})\right.\right. \\
&\quad\left. + (1/g)(K_4 - 1)\left(1 + 3\cos^2\alpha_{2e}\right)\sin(2\phi_{2e})\right] \\
&\quad\left.\times\left[(1 + 1/f)/\left(1 + 3\cos^2\beta_e\right)\right]\left(1/l_e^2\right)\right\} \qquad (9)
\end{aligned}$$

Furthermore, on simplifying the constraint Eq. (1), neglecting higher-order terms, and assuming $\gamma_{ie} = 0$ as yaw maneuver is not possible by changing tether offsets, we get

$$\begin{aligned}
l_t &= l + \sum_{i=1}^{2}\{\hat{a}_{xi}[-\cos(\varphi_{ie})\sin(\eta_e) + \sin(\phi_{ie})\cos(\eta_e)\cos(\alpha_{ie} - \beta_e)] \\
&\quad + \hat{a}_{yi}[\sin(\varphi_{ie})\sin(\eta_e) + \cos(\varphi_{ie})\cos(\eta_e)\cos(\alpha_{ie} - \beta_e)] \\
&\quad + \hat{a}_{zi}[-\cos(\eta_e)\sin(\alpha_{ie} - \beta_e)]\}
\end{aligned}$$

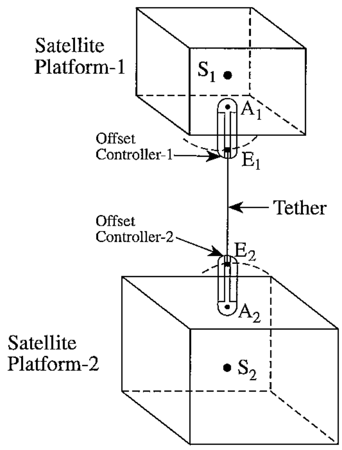

Fig. 2 Geometric configuration of proposed offset controller model.

To have an equilibrium configuration of the system, the tether should be along the local vertical, that is,

$$l_t = l - \hat{r}_{b1} + \hat{r}_{b2}$$

To satisfy the preceding condition, the following tether offset variations corresponding to the desired platform orientations are considered:

$$\hat{a}_{xi} = \hat{r}_{bi}[-\cos(\varphi_{ie})\sin(\eta_e) + \sin(\varphi_{ie})\cos(\eta_e)\cos(\alpha_{ie} - \beta_e)] \qquad i = 1, 2$$

$$\hat{a}_{zi} = -\hat{r}_{bi}\cos(\eta_e)\sin(\alpha_{ie} - \beta_e), \qquad i = 1, 2$$

$$\hat{a}_{yi} = (-1)^i\left\{\hat{r}_{bi}^2 - \hat{a}_{xi}^2 - \hat{a}_{zi}^2\right\}^{\frac{1}{2}}, \qquad i = 1, 2 \qquad (10)$$

The proposed offset controller model is shown in Fig. 2. Thus, for the specific desired fixed platform orientation angle, for example, α_{ie} or φ_{ie}, the tether offsets $\hat{a}_{ki}$, $k = x, y$, should be chosen in accordance with the preceding relations. In case the change of the orientation angles from, for example, $(\alpha_{ie})_0$ or $(\varphi_{ie})_0$ to $(\alpha_{ie})_f$ or $(\varphi_{ie})_f$ is desired, the tether offsets $(\hat{a}_{ki})_0$ initially based on the angle $(\alpha_{ie})_0$ or $(\varphi_{ie})_0$ would have to be changed to attain the values $(\hat{a}_{ki})_f$ corresponding to the final desired orientation $(\alpha_{ie})_f$ or $(\varphi_{ie})_f$. An instantaneous step change in tether offsets from initial to the final values would, however, induce unacceptably large transient oscillations and may even cause instability. To limit the amplitude of transient oscillations, the tether offset change is carried out as follows:

$$(\hat{a}_{ki})_\theta = (\hat{a}_{ki})_0 + [(\hat{a}_{ki})_f - (\hat{a}_{ki})_0]\sin[\theta/(4\tau)] \qquad \text{for} \quad 0 \le \theta/(2\pi\tau) < 1$$

$$= (\hat{a}_{ki})_f \qquad \text{for} \quad 1 \le \theta/(2\pi\tau), \qquad k = x, y \qquad (11)$$

Note that the maximum tether offset required is $|\hat{r}_{bi}|$.

Results and Discussion

To assess the effectiveness of the proposed attitude maneuver strategy, the detailed system attitude response is numerically simulated using Eqs. (1–8) with the following initial conditions:

$$\gamma_{i0} = \alpha'_{i0} = \phi'_{i0} = \gamma'_{i0} = 0, \qquad i = 1, 2$$

$$l_0 = l_e, \qquad \beta'_0 = \eta'_0 = l'_0 = 0$$

For known starting values of variables at each step of numerical integration, we first solve for tether strain using the constraint relation (1). The substitution of the tether strain in Eq. (8) enables determination of the Lagrange multiplier λ. The values of the tether strain and the Lagrange multiplier, thus, explicitly available are utilized to compute the new values of the variables at the end of the step through integration of the set of the differential equations (2–7). The integration is based on NAG routine D02CBF using the variable-step Adams method. The tether offset variations are carried out in accordance with the open-loop policy as given by Eq. (10).

Figures 3–5 show the system response for slewing maneuvers from initial pitch angle $(\alpha_{1e})_0 = -10$ deg, $(\alpha_{2e})_0 = -5$ deg to final pitch angle $(\alpha_{1e})_f = (\alpha_{2e})_f = 0$ deg and initial roll angle $(\phi_{1e})_0 = -10$ deg, $(\phi_{2e})_0 = -5$ deg to final roll angle $(\phi_{1e})_f = (\phi_{2e})_f = 0$ deg, respectively. The platform settles down around the desired pitch and roll angles of 0 deg within less than ± 0.05 and ± 0.09 deg, respectively. The dimensionless length l and its in-plane and out-of-plane swing angles β and η during the roll maneuver are shown in Fig. 5. Note that, in the situation involving roll orientation, the yaw motion gets excited and the efficient yaw control is not achieved.

It is also possible to orient the two platforms at different orientation angles (Figs. 6 and 7). This reorientation would require a judicious choice for the tether offset variations corresponding to the desired final orientation angles α_{1e} or ϕ_{1e} and α_{2e} or ϕ_{2e}.

Next, the effect of tether length l_{t0}, mass distribution parameters K_1 and K_2, radial tether offset $\hat{r}_{bi}$, and parameters τ, C, f, and g on the system attitude response for a typical slewing maneuver from initial pitch angle $(\alpha_{1e})_0 = (\alpha_{2e})_0 = 0$ deg to final pitch angle $(\alpha_{1e})_f = (\alpha_{2e})_f = 15$ deg is investigated. The choice of a dimensionless tether length has a significant effect on platform orientation response. When the tether length considered is 20, the steady-state attitude errors are found to be ± 0.248 deg. While the tether length is increased

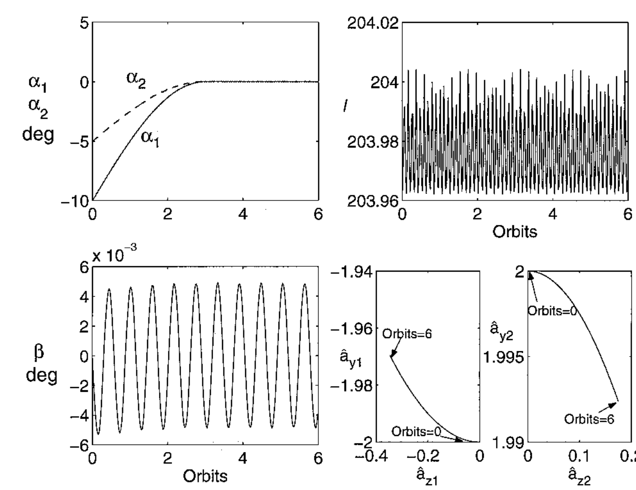

Fig. 3 Typical pitch maneuver system response from $(\alpha_{1e})_0 = -10$ deg, $(\alpha_{2e})_0 = -5$ deg to $(\alpha_{1e})_f = (\alpha_{2e})_f = 0$ deg: $(\alpha_{1e})_0 = -10$ deg, $(\alpha_{2e})_0 = -5$ deg, $(\alpha_{1e})_f = (\alpha_{2e})_f = 0$ deg, $\phi_{10} = \phi_{20} = 0$ deg, $C = 3 \times 10^8$, $K_1 = -0.5$, $K_2 = 0.3$, $l_{t0} = 200$, $|\hat{r}_{bi}| = 2$, $f = 1$, $g = 1$, $\tau = 3$ orbits, $\hat{a}_{xi} = 0$.

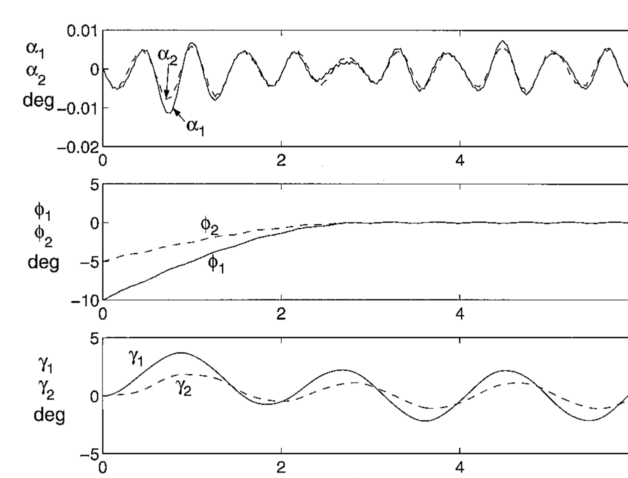

Fig. 4 Typical roll maneuver system response from $(\phi_{1e})_0 = -10$ deg, $(\phi_{2e})_0 = -5$ deg to $(\phi_{1e})_f = (\phi_{2e})_f = 0$ deg: $(\phi_{1e})_0 = -10$ deg, $(\phi_{2e})_0 = -5$ deg, $(\phi_{1e})_f = (\phi_{2e})_f = 0$ deg, $\alpha_{10} = \alpha_{20} = 0$ deg, $C = 3 \times 10^8$, $K_1 = -0.5$, $K_2 = 0.3$, $l_{t0} = 200$, $|\hat{r}_{bi}| = 2$, $f = 1$, $g = 1$, $\tau = 3$ orbits, $\hat{a}_{zi} = 0$.

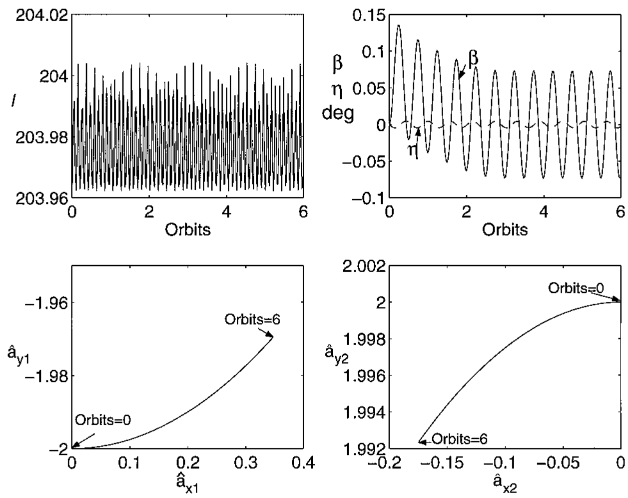

Fig. 5 Nondimensional length l and in-plane and out-of-plane swing angles β and η during roll maneuver from $(\phi_{1e})_0 = -10$ deg, $(\phi_{2e})_0 = -5$ deg to $(\phi_{1e})_f = (\phi_{2e})_f = 0$ deg: $(\phi_{1e})_0 = -10$ deg, $(\phi_{2e})_0 = -5$ deg, $(\phi_{1e})_f = (\phi_{2e})_f = 0$ deg, $\alpha_{10} = \alpha_{20} = 0$ deg, $C = 3 \times 10^8$, $K_1 = -0.5$, $K_2 = 0.3$, $l_{t0} = 200$, $|\hat{r}_{bi}| = 2$, $f = 1$, $g = 1$, $\tau = 3$ orbits, $\hat{a}_{zi} = 0$.

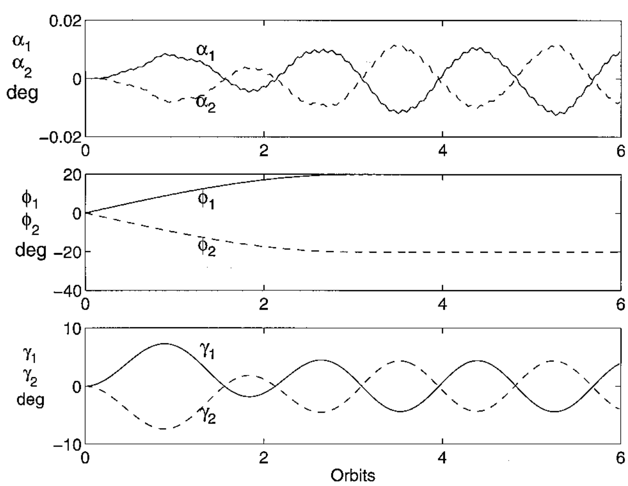

Fig. 6 Typical roll maneuver system response from $(\phi_{1e})_0 = (\phi_{2e})_0 = 0$ deg to $(\phi_{1e})_f = 20$ deg, $(\phi_{2e})_f = -20$ deg: $(\phi_{1e})_0 = (\phi_{2e})_0 = 0$ deg, $(\phi_{1e})_f = 20$ deg, $(\phi_{2e})_f = -20$ deg, $\alpha_{10} = \alpha_{20} = 0$ deg, $C = 3 \times 10^8$, $K_1 = -0.5$, $K_2 = 0.3$, $l_{t0} = 200$, $|\hat{r}_{bi}| = 2$, $f = 1$, $g = 1$, $\tau = 3$ orbits, $\Delta l = l - 204$, $\hat{a}_{zi} = 0$.

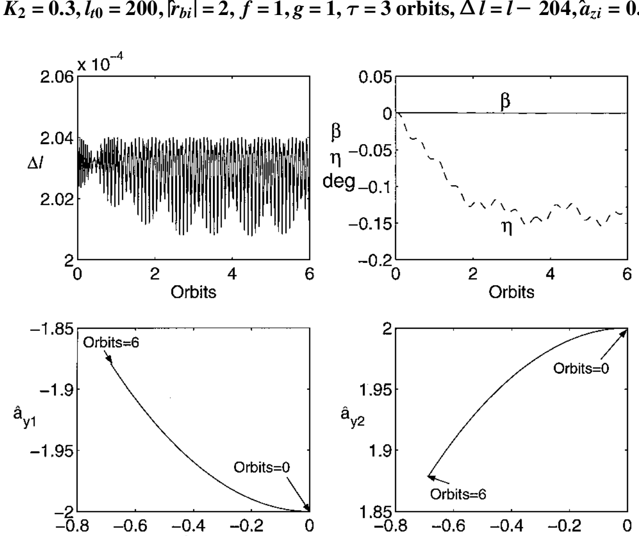

Fig. 7 Nondimensional length l and in-plane and out-of-plane swing angles β and η during roll maneuver from $(\phi_{1e})_0 = (\phi_{2e})_0 = 0$ deg to $(\phi_{1e})_f = 20$ deg, $(\phi_{2e})_f = -20$ deg: $(\phi_{1e})_0 = (\phi_{2e})_0 = 0$ deg, $(\phi_{1e})_f = 20$ deg, $(\phi_{2e})_f = -20$ deg, $\alpha_{10} = \alpha_{20} = 0$ deg, $C = 3 \times 10^8$, $K_1 = -0.5$, $K_2 = 0.3$, $l_{t0} = 200$, $|\hat{r}_{bi}| = 2$, $f = 1$, $g = 1$, $\tau = 3$ orbits, $\Delta l = l - 204$, $\hat{a}_{zi} = 0$.

Table 1 Critical tether length requirements for dual tethered satellite platform systems ($|\hat{r}_{bi}| = 2$, $\tau = 1$ orbit)

System data	Light-weight satellites	Medium weight satellites	Heavy-weight satellites	Lightweight/ medium weight combinations	Shuttle satellite system
m_1, kg	500	1000	10^5	1000	10^5
m_2, kg	500	1000	10^5	500	5×10^4
I_{x1}, kg-m^2	100	500	10^7	500	10^7
I_{x2}, kg-m^2	100	500	10^7	100	10^6
I_{y1}, kg-m^2	150	1000	10^7	1000	10^7
I_{y2}, kg-m^2	150	1000	10^7	150	10^6
I_{z1}, kg-m^2	75	500	8×10^6	500	8×10^6
I_{z2}, kg-m^2	75	500	8×10^6	75	8×10^5
$(L_{t0})_{cr}$, m	6.8	10.7	150.0	15.0	212.1

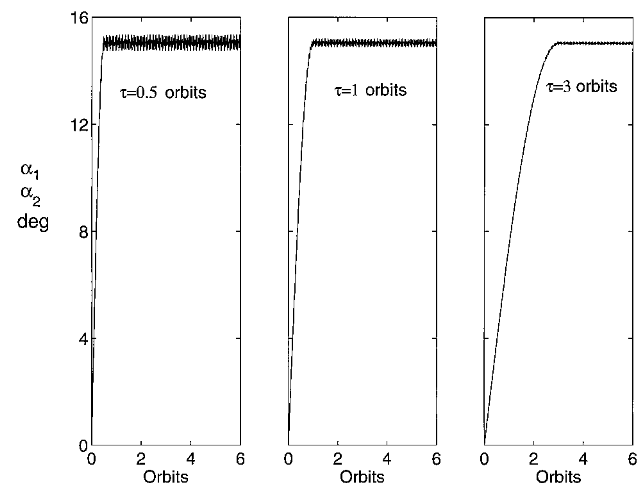

Fig. 8 Typical system attitude maneuver response as affected by τ: $(\alpha_{1e})_0 = (\alpha_{2e})_0 = 0$ deg, $(\alpha_{1e})_f = (\alpha_{2e})_f = 15$ deg, $\phi_{10} = \phi_{20} = 0$ deg, $C = 3 \times 10^8$, $K_1 = -0.5$, $K_2 = 0.3$, $l_{t0} = 200$, $|\hat{r}_{bi}| = 2$, $f = 1$, $g = 1$, $\hat{a}_{xi} = 0$.

from $l_{t0} = 20$ to 200, the steady-state attitude errors decreases from ± 0.248 to ± 0.097 deg. In general, it is observed that an increase in the tether length results in progressively reduced librational amplitudes in platform orientation errors. However, below a critical tether length $(l_{t0})_{cr}$ the attitude errors becomes very large, and further decrease in tether length results in instability of the system. The critical tether length requirements for various combination of dual satellite platforms based on the numerical simulation of the satellite attitude responses are listed in Table 1. The platform mass distribution parameters K_1 and K_2 appear to have little effect on platform orientation characteristics. It is observed that, even for the inherently most unstable platform mass distributions, the response is virtually the same as that for other stable or unstable combinations. The effect of the most adverse platform mass distributions is only one of slightly increased maximum platform orientation errors.

The influence of the dimensionless radial tether offset $\hat{r}_{bi}$ on system orientation performance is studied next. As $|\hat{r}_{bi}|$ is increased from 0.1 to 2.0, the steady-state attitude errors decreases from 1.309 to 0.097 deg. In general, increase in $|\hat{r}_{bi}|$ results in greater precision in orientation. When $|\hat{r}_{bi}|$ selected is very small, the librational amplitudes is relatively large and unacceptable.

The parameter τ is found to have significant influence on the resulting steady-state orientation errors (Fig. 8). The rapid changes in the tether offsets signified by the relatively small values of τ may lead to unacceptably large orientation errors and instability. When the offsets $\hat{a}_{ki}$, $k = x, y$, are varied slowly as signified by larger τ values, the system settles down close to the desired equilibrium configuration. The larger the value of τ, the smaller are amplitudes of oscillations around the nominal equilibrium orientation of each platform.

There is no effect of varying the parameter C on system attitude maneuver response. However, the amplitude of tether longitudinal oscillations increases from 6×10^{-9} to 2.6×10^{-5} as C is varied from 10^8 to 10^4. This increase in amplitude may be due to the flexibility of the tether. The changes in the parameters f and g have no observed effect on system maneuver performance.

Conclusions

Attitude maneuvering of a dual satellite platform through change of tether offsets is presented. An open-loop control law for changing tether offsets to achieve desired orientation of the platforms is proposed and developed. The exact numerical integration of the governing nonlinear equations of motion establishes the feasibility of the proposed concept. The small tether length as well as offset variation on the order of a fraction of a meter may suffice to provide the desired orientation of platforms. However, the system has the limitation of controlling yaw excitation in case of roll maneuver. The nearly passive nature of the proposed mechanism using short tethers makes the concept particularly attractive for future space missions.

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